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Noise reduction in Eden models: II. Surface structure and intrinsic width

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Abstract. We suggest a picture of the intrinsic surface width in Eden models and show that it is a major source of corrections to scaling of the surface roughness. Using the multiple-hit noise reduction method we can control the intrinsic width and thereby improve the scaling behaviour systematically as we demonstrate in detailed calculations on the square lattice. We calculate the number of excess perimeter sites as a function of time and find that its asymptotic value decays with a power law as a function of increasing hitting number. Substrate effects and anisotropy become more apparent if noise reduction is applied.

1. Introduction

The structure and fluctuations of an interface between two phases has been a long-standing unsolved problem of statistical mechanics (Smoluchowsky 1908, Rowlinson and Widom 1982). Concepts, such as the intrinsic surface width, are lacking a clear definition and have been subject to extensive discussions (Huse *et al* 1985, Percus and Williams 1986). Simple interfacial growth models (Herrmann 1986, Kertész 1987) are expected to contribute to the understanding of these problems.

In addition, there are non-equilibrium growth phenomena which seem to be adequately treated by such models (Leamy *et al* 1980). They are, however, interesting in their own right too, because of the complex behaviour of the developing surface. The intimate relation of these models to other problems of theoretical physics, such as shock wave propagation described by the Burgers equation or polymer statistics (Kardar *et al* 1986, Stauffer and Jan 1987) on the one hand and spin models (Meakin *et al* 1986b, Plischke *et al* 1987) on the other, has motivated further the activity in this field.

The Eden model (Eden 1961) is a paradigm for stochastic growth of compact clusters exhibiting non-trivial scaling of the surface. Despite its simplicity (a cluster is grown on a lattice by incorporating its current perimeter sites randomly) it took considerable effort to understand its main features at least in two dimensions (Stauffer 1987). Another related model is ballistic aggregation (Vold 1963) which probably belongs to the same universality class as the Eden process (cf Meakin *et al* 1986b).

Eden clusters are compact in all dimensions (Dhar 1985): the number of particles N in the cluster is proportional to R^d where R is the linear size of the cluster and d

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is the Euclidean dimension. The surface, however, is rough, i.e. the surface width w diverges with increasing system size. The dynamic scaling of this width is non-trivial (Plischke and Rácz 1984) and can be given for clusters grown on a substrate of size L^{d-1} as (Family and Vicsek 1985)

$$\begin{aligned} & w \approx L^{\alpha} f(t/L^{z}) \\ & f(x) \to f_{\infty} \qquad \text{for } x \to \infty \\ & f(x) \approx c x^{\beta} \qquad \beta = \alpha/z \qquad \text{for } x \to 0 \end{aligned} \tag{1}$$

where $w^2 = \langle (r - \langle r \rangle)^2 \rangle$, r being the height of a perimeter site above the substrate and $t = N/L^d$ is the time proportional to the cluster size. α and β are critical exponents, the averaging $\langle \ldots \rangle$ is extended over all perimeter sites and f_{∞} and c are constants.

Recent theoretical work (Kardar *et al* 1986, Kardar and Zhang 1987) based on a Langevin equation approach to surface evolution (Edwards and Wilkinson 1982) and the exact solution of a modified Eden model (Dhar 1987) suggest $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ in two dimensions. Numerical results (Jullien and Botet 1985, Plischke and Rácz 1985, Hirsch and Wolf 1986, Zabolitzky and Stauffer 1986, Wolf and Kertész 1987a) support this prediction.

The situation in higher dimensions is less clear. Some assumptions on the polymer analogue of the problem suggest superuniversal (dimension-independent) exponents (Kardar *et al* 1986, Kardar and Zhang 1987) while Edwards and Wiklinson (1982) and Meakin *et al* (1986b) predict $\alpha = 0$ for $d = 3$, a value obtained for roughening in capillary wave theory (Jasnow 1985). However, simulation results for ballistic aggregation (Meakin *et al* 1986b) and the Eden model (Wolf and Kertész 1987b) contradict both conjectures and support a recently proposed scaling law between the exponents (Meakin *et al* 1986b, Kardar and Zhang 1987, Krug 1987): $z = 2 - \alpha$.

The numerical investigation of the scaling behaviour of the surface in Eden growth is hindered by anisotropy and strong corrections to scaling. The difficulties due to anisotropy can be overcome if the cluster is grown on a flat substrate. The corrections are a more serious problem. The multiple-hit (noise reduction) method turned out to be a powerful tool in order to reach the asymptotics with relatively small computing effort (Wolf and Kertész 1987a, hereafter referred to as I). Using this method we calculated the exponents in three and four dimensions (Wolf and Kertész 1987b) and found $\alpha \approx 1/d$ and $z \approx 2 - \alpha$.

Noise reduction not only improves the scaling behaviour but also gives a better insight into the nature of the corrections to scaling, the substrate effects and the influence of lattice anisotropy. The aim of this paper is to study the latter points by presenting further results on the two-dimensional model and to contribute to the understanding of the structure of surfaces.

One of the basic concepts in the theory of surfaces is the intrinsic width (Rowlinson and Widom 1982). In the case of the Eden model noise reduction enables us to separate the contributions to the total surface width originating from the intrinsic width and from long-wavelength fluctuations, respectively. Furthermore the asymptotic behaviour of these contributions can be described by a general scaling ansatz which incorporates the noise reduction. This will be elaborated in § 2.

Wolf (1987) pointed out the importance of perimeter density in the characterisation of the surface. We show in § 3 how this quantity depends on noise reduction. Also, specific subsets of the perimeter will be discussed.

Reducing the noise slows down the development of fluctuations. Therefore the coherence due to the substrate is observable for longer than in the original model: the growth proceeds layerwise in the early stage and consequently the anisotropy is stronger. We give an account of these effects in § 4.

In § 5 we summarise and discuss our results. An appendix contains the calculation of the time dependence of the width and perimeter density in layerwise growth.

2. The surface width

2.1. The model

We consider the two-dimensional Eden growth process on a flat substrate of size L with noise reduction as introduced in I. The model is defined in the following way. At the beginning only the substrate sites are occupied. In the usual Eden model one of the perimeter sites (empty sites which are nearest neighbours to at least one occupied site) is chosen at random and is added to the cluster. Instead of doing so we count the number of trials for a given growth site and occupy it only if this number reaches a prescribed value m called the noise reduction parameter or hitting number. The $m = 1$ case is obviously the original Eden model. Figure 1 presents a couple of snapshots of noise-reduced Eden clusters.

The above procedure represents model A in the terminology of Jullien and Botet (1985); we also carried out calculations with model C where first an occupied site on the boundary (already belonging to the cluster) is picked randomly and then one of its empty neighbour sites is filled by chance. In this case we count the trials on the boundary sites.

In order to study the effect of anisotropy we consider geometries where the growth proceeds in the (1, 0) and (1, 1) directions. Perpendicular to the growth we implemented periodic (or more precisely helical) boundary conditions. The time is identified with N/L where N is the total number of sites in the cluster grown on the substrate. Most of our data presented here are for model A and growth direction (1, 0) and we shall always emphasise it if we deal with a different situation. The number of independent runs times the number of particles in the cluster was typically between 10^8–10^9. The computations were carried out on a Cyber 76.

2.2. Intrinsic surface width and corrections to scaling

The considerable improvement in the scaling behaviour of the Eden model due to noise reduction (I) can be traced back to the structure of the surface. There are two contributions to the width of the surface (Zabolitzky and Stauffer 1986): the long-wavelength fluctuations which are responsible for the scaling described by (1) and the superimposed L-independent intrinsic width. In I we argued that the intrinsic width characterising the internal structure of the surface is due to holes, overhangs and high steps. Since noise reduction suppresses the formation of these deviations from single-step SOS (solid-on-solid, see, e.g., Huse *et al* (1985)) configurations the corrections to scaling caused by the intrinsic width are decreasing, whereas the long-wavelength fluctuations are still present.

One would like to construct an ansatz which expresses these assumptions in a mathematical form based upon the above considerations. For the description of the

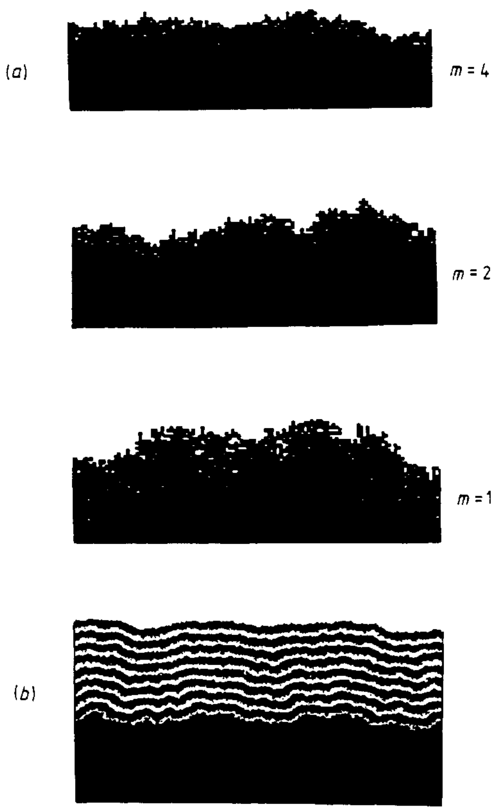

Figure 1. (*a*) Snapshots of noise-reduced Eden clusters grown on a substrate of $L = 160$, each containing 25 000 particles, for different values of the hitting parameter m. We cut the lower parts where no holes were present any longer. (*b*) Time evolution of a cluster growing at first with $m = 1$. At $N_0 = 10^5$ noise reduction with $m = 16$ is switched on. The surface configuration is shown for $N = N_0 + 6250i$, $i = 0, \ldots, 16$. This picture illustrates lateral growth, decay of short-wavelength fluctuations present at $N = N_0$ and precursors of singularities ('shock waves') as described by Kardar *et al* (1986). ($L = 625$.)

contribution of the intrinsic width w_i we suggest the following equation:

$$w^2(L, t) = [aL^{\alpha} f(t/L^z)]^2 + w_i^2 \qquad (2)$$

where a and w_i are independent of L. The coefficient a for the amplitude is normalised to approach unity as $m \to \infty$. w_i is the intrinsic width of the surface region. Both a and w_i are expected to reach stationary values after an L-independent saturation time. Thus for large L and t they can be regarded as constants. The quadratic summation in (2) arises naturally if w is regarded as the width of the convolution of two independent Gaussian distributions, one describing the intrinsic width and the other the long-wavelength fluctuations. However, we consider (2) as an ansatz to be tested by our data. Obviously, w_i is a source of corrections to scaling.

Figure 2 illustrates how well ansatz (2) fits the data in the limit of $t/L^z \to \infty$. (Here we made use of the result that in two dimensions $\alpha = \frac{1}{2}$ (Kardar *et al* 1986, Zabolitzky

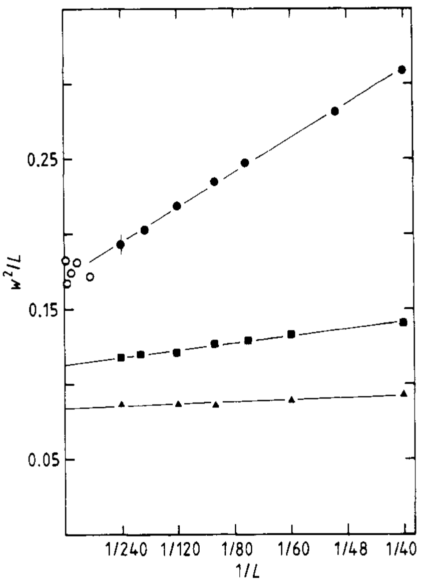

Figure 2. Plot of w^2/L against $1/L$ for $m=1$ (●, ○), 2 (■) and 4 (▲). According to (2) the slopes can be identified with the intrinsic width w_i^2. The intersections with the vertical axis are the amplitudes of the long-wavelength fluctuations. Data for $m=1$ are taken from Wolf (1987) ($L \leq 240$) and Zabolitzky and Stauffer (1986) ($L > 240$).

and Stauffer 1986).) A similar plot using the linear superposition ($w = aL^{1/2}f + w_i$) has a worse fit to the same data.

The slopes of the straight lines in figure 2 are the different values obtained for w_i^2 for $m=1$, 2 and 4. The intrinsic width is indeed suppressed by increasing m: the slopes are decreasing. The slopes for higher values of m are smaller than our error bars; from the three values available

$$w_i \approx 2.3/m \tag{3}$$

seems to hold approximately.

The intercepts of the lines with the vertical axis correspond to the amplitudes of the long-wavelength fluctuations for various m values. Figure 3 shows that we can fit the m dependence of these intercepts quite well by

$$a^2 \approx (1 + 2.3/m) \qquad f_\infty^2 \approx 0.052. \tag{4}$$

(It is worth mentioning that, for the amplitude analogous to f_∞^2, the value 0.07 can be obtained from data by Meakin *et al* (1986b) in a related single-step growth model.) Combining (2)-(4) for $t \to \infty$ we get the following expression:

$$w^2(t \to \infty, L, m) = 0.052(1 + 2.3/m)L + (2.3/m)^2 \tag{5}$$

which leads, for $m=1$ and large L, to $w \approx 0.414L^{1/2}(1 + 15/L)$ while Zabolitzky and Stauffer (1986) obtained from their large-scale calculation $w \approx 0.42L^{1/2}(1 + 18/L)$ purely empirically.

In our interpretation both the intrinsic width and the term $a-1$ originate from holes, overhangs and steps higher than unity. Among these contributions the high steps are most important because holes and overhangs are built up in processes of higher order in noise in the sense that a high step is already needed for forming an overhang and a hole is an overhang closing at least two higher steps. In § 4 it will be shown that the number of perimeter sites in steps higher than unity vanishes as $1/m$. Therefore it seems plausible that $a-1$ and w_i are also proportional to $1/m$.

2.3. Noise reduction and dynamic scaling

So far we have considered the $t\to\infty$ limit where the width saturates for a given L. However, noise reduction not only influences this asymptotic; it has very important effects on dynamic properties as well. Two observations can be made from a log-log plot of the width against time for various values of m (figure 4). For sufficiently large m one can recognise fairly linear parts at intermediate times between a short-time regime dominated by the substrate and the long-time stationary regime. The second

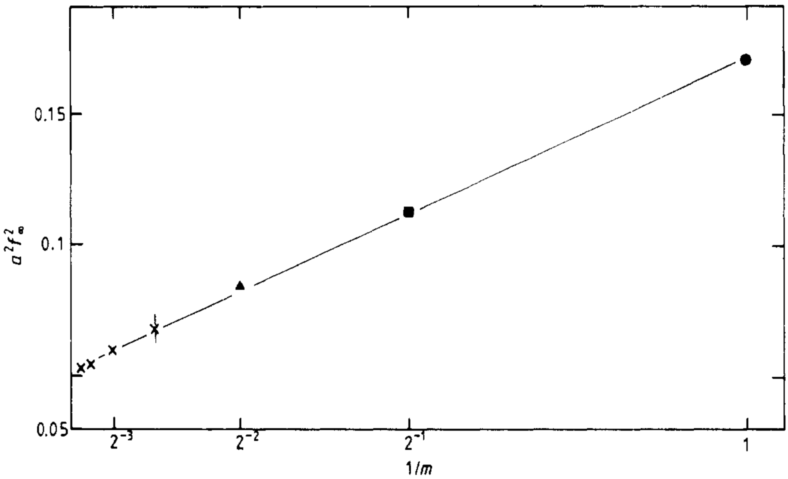

Figure 3. The effect of noise reduction on the amplitude of the long-wavelength fluctuations shown on a w^2/L against $1/m$ plot. Filled symbols correspond to an extrapolation to $L\to\infty$ (cf figure 2); ×, $L=240$; the slope is 2.3 ± 0.1.

observation is that noise reduction introduces a new timescale: the curves are shifted towards larger cluster sizes for increasing m. Including the m dependence into the general scaling ansatz (2) one has to write

$$w^2(t, L, m) = [a(m)L^\alpha f(tL^{-z}m^{-\xi})]^2 + w_i^2(m) \tag{6}$$

with $a(m)$ and $w_i(m)$ for model A and growth direction $(1,0)$ given by (3) and (4), respectively.

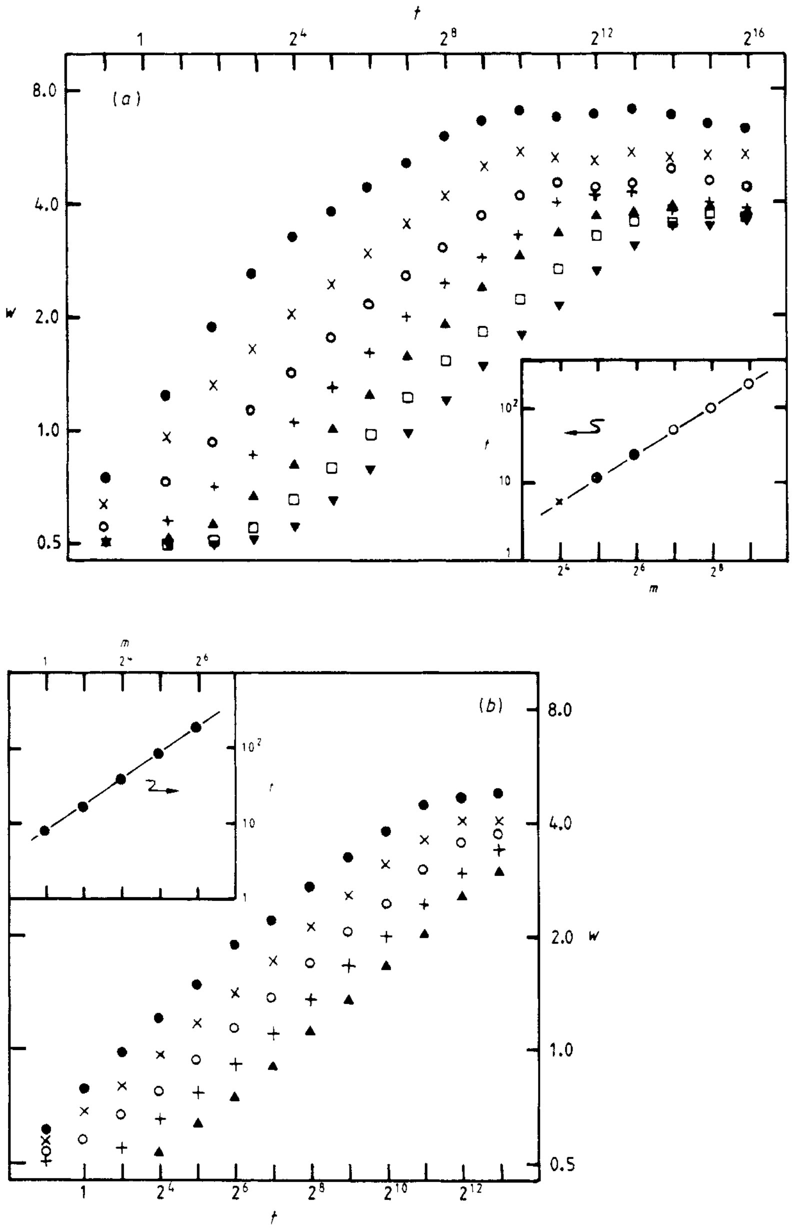

Figure 4. Noise reduction introducing time rescaling. (*a*) Model A with $L = 240$, for a sequence of $m = 2^i$, $i = 0, 2, \ldots, 6$, m increasing from the left to the right. Straight parts on this log w against log t plot mark the dynamical scaling region. The insert shows the determination of the exponent ξ (see (7)) by using the data for $w = 1.2$, $L = 240$ ($\times$) and $L = 960$ ($\bigcirc$). A similar plot for model C is presented in (*b*) with $L = 240$ and $i = 0, \ldots, 4$. Here the insert is also based on data at $w = 1.2$.

The exponent ξ is determined from a log-log plot of t against m at a fixed w in the limit of large m where one can set $a=1$ and $w_i=0$ (see the inserts in figure 4). The slope ξ is independent of L and w if chosen from the dynamical scaling region. For model A we get $\xi=1.05\pm0.1$. In figure 4(b) we present data for model C from which we obtain $\xi=1.1\pm0.1$ (ξ is possibly universal for models A and C). We see that, with noise reduction, the approach to the scaling region in models A and C is not as different as for $m=1$ (Jullien and Botet 1985, Meakin *et al* 1986a).

For small m the intrinsic width leads to strong corrections to scaling in the dynamical scaling region where we get, by combining (1) with (6),

$$w^2(t,m)=c^2[a(m)(t/m^{\xi})^{\beta}]^2+w_i^2(m). \tag{7}$$

Figure 5 shows the evaluation of our data according to (7). In the region where the scaling (6) is valid we get a straight line in the $w^2-w_i^2(m)$ against $[a(m)(t/m^{\xi})^{\beta}]^2$ plot where $a^2(m)$ is taken from (4). The slope determines $c^2=0.52$.

The ansatz (7) leads to the effective exponent $\beta_{\text{eff}}=\mathrm{d}(\ln w)/\mathrm{d}(\ln t)$

$$\beta_{\text{eff}}=\beta[1+(w_i(m)m^{\xi\beta})^2/(a(m)ct^{\beta})^2]^{-1} \tag{8}$$

or, for large t, using the numerical values obtained for model A with growth direction (1, 0):

$$\beta_{\text{eff}}=\tfrac{1}{3}[\![1-\{2.3m^{-2/3}[(1+2.3/m)0.52t^{1/3}]^{-1}\}^2/2]\!] \tag{9}$$

for the t and m dependence of β_{eff} where the value $\beta=\frac{1}{3}$ is used (Kardar *et al* 1986,

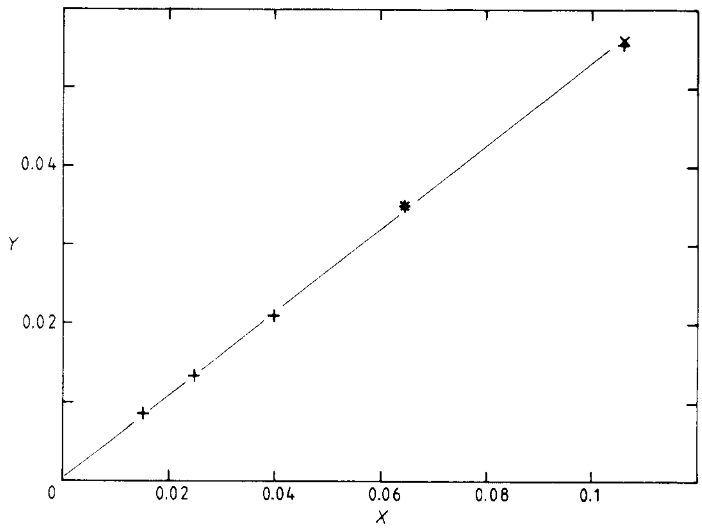

Figure 5. Determination of the coefficient c^2 from the $Y=w^2-w_i^2$ against $X=[a(t_0/m)^{1/3}]^2$ plot (cf (7)). The values of w are taken for $L=240$, $t_0=256$ (×) and $L=960$, $t_0=950$ (+). From the slope we obtain $c^2\approx0.518$.

Zabolitzky and Stauffer 1986, I). For $m = 1$ we can compare our result (9) with the data from Zabolitzky and Stauffer (1986). Equation (9) explains why for large cluster sizes $\beta_{\rm eff}$ approaches $\beta = \frac{1}{3}$ from below. Together with the substrate-induced value of $\beta_{\rm eff} = 0.5$ at small times this implies the occurrence of a minimum in $\beta_{\rm eff}(t)$.

We can also see from (8) that the correction to β in $\beta_{\rm eff}$ due to the intrinsic width is vanishing as $m^{2(\xi\beta-1)}$. This explains our observation in I that noise reduction improves dynamical scaling behaviour. We obtained similar behaviour for dimensions higher than two (Wolf and Kertész 1987b).

3. Perimeter density

The number of perimeter sites $N_{\rm p}$ per unit length is an important characteristic of the surface which we denote by $v = N_{\rm p}/L$. We have investigated how it is influenced by noise reduction. Here we only discuss our results for model A and growth direction (1, 0), where its behaviour is particularly transparent. Some remarks concerning the dependence on the growth direction and differences between models A and C will be made in the next section.

Similar to the intrinsic width, the perimeter is also related to the high steps, holes and overhangs. For surface configurations without these features $N_{\rm p} = L$. Consequently, the excess perimeter density $v - 1$ is expected to approach zero for increasing hitting number m.

As for $m = 1$ (Hirsch and Wolf 1986) the perimeter density is independent of the substrate length L (figure 6). Therefore, for large L it reaches a stationary value much earlier than the width (Wolf 1987). Again we find that noise reduction shifts the stationary regime towards later times (cf the function f in (6)). However, a simple

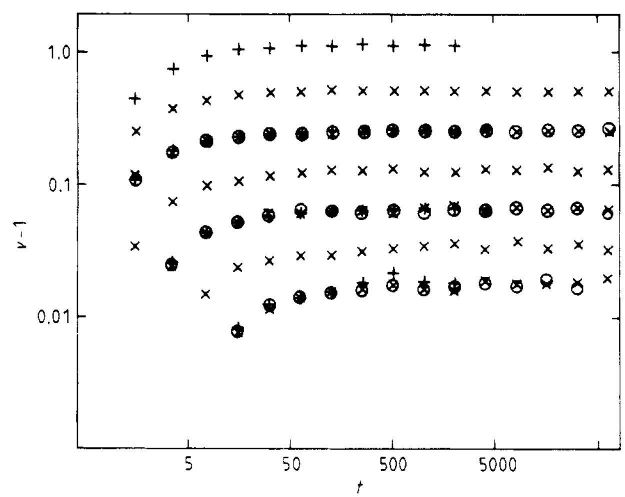

Figure 6. Time dependence of excess perimeter density $v - 1$ for $L = 120$ (+), 240 (×), 960 (○). m varies from the top curve to the bottom curve as 2^i, $i = 0, 1, \ldots, 6$.

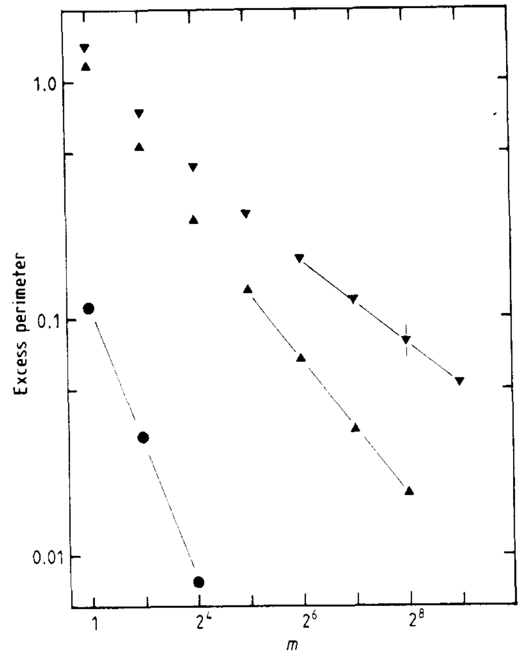

Figure 7. Excess perimeter for growth direction $\theta = \pi/4$ $(v - 1/\sqrt{2})$ (▼) and for $\theta = 0$ $(v - 1)$ (▲), and density of perimeter sites in overhangs (N_0/L) for $\theta = 0$ (●) as functions of m on a log-log plot.

quantitative analysis similar to (6) is not possible here because of the substrate effects, to be discussed in the next section.

Figure 7 shows a log-log plot of the asymptotic value of $v - 1$ against m. For m between 8 and 64 we find that the excess perimeter density decays as

$$v - 1 \sim m^{-0.95 \pm 0.05} \tag{10}$$

confirming that noise reduction suppresses the extra perimeters contained in overhangs, holes and high steps.

The different contributions to the excess perimeters are expected to be selectively sensitive to the noise reduction. To illustrate this we determined the density of overhangs N_0/L. Figure 7 shows that $N_0(m)$ decreases with an exponent roughly twice as large as that for the total excess perimeter. The reason is that overhangs can only be created at high steps. Thus they are more strongly suppressed than the high steps. The creation of holes is a process of even higher order as it requires the closing of overhangs but in the statistics this may be compensated by lifetime effects.

4. Substrate effects and anisotropy

In the limit $m \to \infty$ the Eden cluster (model A) grows layer by layer. A new generation of perimeter sites becomes active only after all sites of the preceding generation have been occupied. This is also true for model C for $\theta = 0$ (θ is the angle between the orientation of the growth and the direction with Miller indices (1, 0), $0 \leq \theta \leq \pi/4$). However, for $\theta > 0$ there exist occupied sites with more than one empty neighbour which are selected by chance, so that there is somewhat more room for randomness in model C.

As a consequence of the layerwise growth ($m \to \infty$) in model A the surface width for all θ and the perimeter density for $\theta > 0$ have oscillatory behaviour. If τ is the fraction of sites occupied in one layer, the following expressions can be obtained:

$$v(\tau, \theta = 0) = 1 \tag{11a}$$

$$w(\tau, \theta = 0) = [\tau(1-\tau)]^{1/2} \tag{11b}$$

$$v(\tau, \theta = \pi/4) = [1 + \tau(1-\tau)]/\sqrt{2} \tag{11c}$$

$$w(\tau, \theta = \pi/4) = [T(1-T)]^{1/2}/\sqrt{2} \tag{11d}$$

with $T = (1-\tau)/[1+\tau(1-\tau)]$. The derivation of (11) is given in the appendix.

If m is finite, random fluctuations will lead to phase incoherence of these oscillations in different clusters of equal size and in the average over many clusters they will be washed out. However, for not too large times, one can still observe them. Figure 8 shows the width for $\theta = 0$ as a function of the scaled time t/m^{ξ} for different m. We used $\xi = 1$. The upper curve corresponds to half-integer t (i.e. $\tau = \frac{1}{2}$) where, according

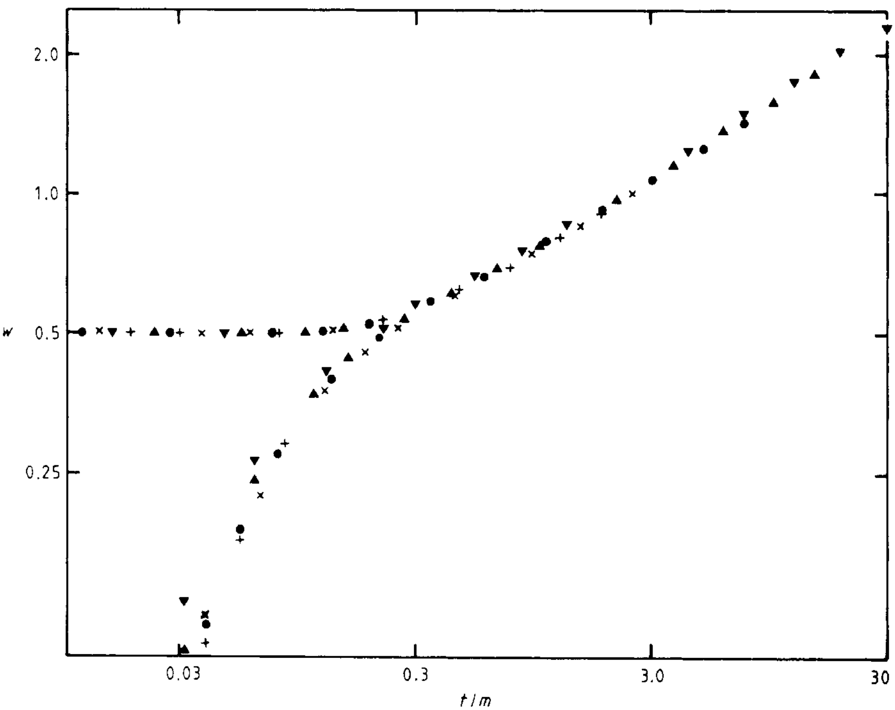

Figure 8. Oscillations of the surface width during the early stage of the growth on a substrate of length $L = 960$. Upper values, half-integer times; lower values, integer times. Time is scaled with noise reduction parameter $m = 32$ (▼), 64 (▲), 128 (●), 256 (×), 512 (+).

to (11*b*), the width is maximal. The lower curve represents the values for integer t. The width oscillates between the two curves as a function of t.

Some further interesting features of these substrate effects should be emphasised: they are independent of the size of the substrate and survive for longer the larger m is. In other words, increasing m increases the dynamical correlations in the system as the information about a configuration is maintained in layerwise growth.

For the excess perimeter $v-1$ in the $\theta=0$ direction we have also observed oscillatory behaviour for hitting numbers $m>16$. It extends to the stationary region, where our resolution of data becomes insufficient.

These oscillations are surprising in view of (11*a*) but can be explained in the following way. Due to the finiteness of m the surface develops a certain number of regions tilted with respect to the $\theta=0$ orientation. If m is large enough to ensure almost layerwise growth, these tilted regions produce oscillations in v analogous to (11*c*).

Using large noise reduction, the oscillations of the perimeter density in the $\theta=\pi/4$ direction are described for short times by (11*c*). For long times the perimeter density saturates. However, for $m>64$ we observe saturation values below $7/6\sqrt{2}$, the average over one period (the uppermost curve in figure 7). This is a subtle effect due to dynamic correlations built up for large m: if a perimeter is occupied in the case of diagonal growth, two new perimeters are born next to it. Therefore the assumption of independent random occupation leading to (11) will not be valid and clustering within a layer occurs which suppresses v.

Figure 7 also shows that the excess perimeter density decays more slowly for $\theta=\pi/4$ than for $\theta=0$ (10). From the data between $m=16$ and $m=64$ we obtain $v-1/\sqrt{2}\approx m^{-0.58\pm0.07}$.

In Dhar (1986) and Wolf (1987) it was shown that, for $m=1$, v has the meaning of a growth velocity. Its dependence on the orientation θ in the substrate geometry determines the shape of Eden clusters grown from a single seed. This raises the question whether the oscillations in (11*c*) have any influence on the shape of the Eden cluster in the limit $m\to\infty$. The answer is 'no'. By similar arguments to Wolf (1987) one can see that for $m>1$ the growth velocity is the number of 'ripe' perimeter sites where the counter is at a value $m-1$. In the diagonal directional for layerwise growth there are never more than $1/\sqrt{2}$ ripe perimeter sites and therefore the cluster will be diamond-shaped.

5. Discussion and summary

In this paper we have given a detailed study of the square lattice noise-reduced Eden models A and C. The method of noise reduction was originally introduced for the case of Laplacian growth by Tang (1985) and Szép *et al* (1985) and was successfully used to study the asymptotic shape of the clusters (Kertész and Vicsek 1986, Nittmann and Stanley 1986). In I we demonstrated that the application of the multiple-hit noise reduction algorithm saves computer time and memory in the investigation of the Eden model since it increases the region where dynamic scaling can be observed. The Eden model in three and four dimensions exhibits similar behaviour (Wolf and Kertész 1987b).

According to our picture the improvement in the scaling behaviour is due to the suppression of the intrinsic width by noise reduction. This is expressed in equation

(6) where the effect of noise on the amplitude of the scaling function f and on the timescale can also be seen. Due to noise reduction one can distinguish two characteristic times, m^ξ and $L^z m^\xi$. Accordingly $t \ll m^\xi$ is the regime of strong substrate effects as displayed in figure 8, $m^\xi \ll t \ll L^z m^\xi$ is the dynamical scaling region illustrated in figure 4 and finally $t \gg L^z m^\xi$ is the stationary regime where the surface width reaches its asymptotic value. For $t \ll L^z m^\xi$ the width of the surface region does not depend on L.

Our data displayed in figures 2-5 support assumption (6) and in the text we presented the numerical values for the coefficients (3) and (4) and the exponent ξ. Figures 4 and 5 demonstrate that introducing noise reduction improves the dynamic scaling considerably (see also results in I) and that important corrections to scaling stem from the intrinsic width. Of course, size-dependent corrections should also be taken into account (Zabolitzky and Stauffer 1986). According to I they show up in systematic trends.

We think that the scaling form (6) and its numerical verification is the main result of the present paper. The ansatz given there might well work for other systems such as percolation (Franke 1980) or thermal models as well. Since the intrinsic structure of the surface is attracting increasing interest from the point of view of theoretical (Huse *et al* 1985) and experimental (Beaglehole 1987) considerations and also of morphology (Chowdhury 1988) we think that an analysis analogous to the present one for a Hamiltonian system would be useful. In this case we expect the temperature to play the role of $1/m$.

The limit $m \to \infty$ is subtle in the sense that, for $m = \infty$, there is no roughening at all since noise is necessary for developing the long-wavelength fluctuations. This suggests that—in a renormalisation group terminology—$1/m$ is a relevant parameter, i.e. we get the same exponents α and z for every value of m, except for $m = \infty$.

We have seen that noise reduction introduces a time rescaling by shifting the curves towards larger sizes both for the width and for the perimeter density. Therefore, in order to take advantage of our method, one has to choose an optimal value for m: large enough for reducing the intrinsic width but still small enough to reach the scaling region within a reasonable computing time. In Wolf and Kertész (1987b) we used $m = 8$ for $d = 3$ and 4.

Noise reduction turned out to reduce the excess perimeter in the same way as the intrinsic width. In fact, since both quantities receive the main contributions from steps higher than unity, the excess perimeter density seems to be a good measure of the intrinsic width. The nearly identical $1/m$ dependence of these quantities points in this direction (see (3) and (10)). We think that this concept is worth testing in other systems, too.

In conclusion, we have shown that the application of noise reduction is very useful not only because it improves the scaling behaviour of the surface width but also because it helps in clarifying basic concepts for the understanding of the structure of surfaces.

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Appendix

We give here the derivation of (11c, d). Let τ denote the filling of the first monolayer on a substrate parallel to the diagonal of a square lattice. The minimal number of perimeter sites is equal to the number of adsorption sites of the substrate:

$$X = L/\sqrt{2}. \tag{A1}$$

For $m \to \infty$, supposing that there are no correlations among the particles (i.e. for $t \ll m^{\xi}$), the probability that an empty site is the left neighbour of an occupied one is $\tau(1-\tau)$. Whenever this is the case there is one perimeter site in addition to X, so that the total average number is

$$N_{\rm p} = X[1+\tau(1-\tau)]. \tag{A2}$$

With (A1) one obtains the number of perimeter sites per unit length (11c).

The average distance of the perimeter sites from the substrate is

$$\langle r \rangle = X[(1-\tau)+2\tau+2\tau(1-\tau)]/N_{\rm p}\sqrt{2} \tag{A3}$$

and similarly

$$\langle r^2 \rangle = X[(1-\tau)+4\tau+4\tau(1-\tau)]/N_{\rm p}2. \tag{A4}$$

Hence $w^2 = \langle r^2 \rangle - \langle r \rangle^2 = T(1-T)/2$ with T given below (11d). The derivation of (11a) and (11b) is simpler and follows similar lines.

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